

Light mass galileons: Cosmological dynamics, mass screening and observational constraints

Amna Ali,¹ Radouane Gannouji,² Md. Wali Hossain,³ and M. Sami^{4,3}

¹*Astroparticle Physics and Cosmology Division and Centre for Astroparticle Physics,
Saha Institute of Nuclear Physics, 1/AF Bidhannagar, Kolkata 700 064, India*

²*Department of Physics, Faculty of Science, Tokyo University of Science,
1-3, Kagurazaka, Shinjuku-ku, Tokyo 162-8601, Japan*

³*Centre for Theoretical Physics, Jamia Millia Islamia, New Delhi-110025, India*

⁴*Kobayashi-Maskawa Institute for the Origin of Particles and the Universe,
Nagoya University, Nagoya 464-8602, Japan*

In this paper, we examine the cosmological viability of a light mass galileon field consistent with local gravity constraints. The minimal, $L_3 = \square\phi(\partial_\mu\phi)^2$, massless galileon field requires an additional term in order to give rise to a viable ghost free late time acceleration of Universe. The desired cosmological dynamics can either be achieved by incorporating an additional terms in the action such as (L_4, L_5) – the higher order galileon Lagrangians or by considering a light mass field *à la* galileon field potential. We analyse the second possibility and find that: (1) The model produces a viable cosmology in the regime where the non-linear galileon field is subdominant, (2) The Vainshtein mechanism operates at small scales where the non-linear effects become important and contribution of the field potential ceases to be significant. Also the small mass of the field under consideration is protected against strong quantum corrections thereby providing quantum stability to the system.

I. INTRODUCTION

Modified theories of gravity pose a serious alternative to dark energy, an exotic cosmic fluid, needed to account for late time cosmic acceleration in the framework of standard lore[1–8]. The task of building an alternative to Einstein’s theory is very challenging as the latter fits with the observation with a great accuracy locally thereby a large scale modification is either felt locally or the framework gets reduced to Λ CDM. A viable alternative theory of gravity should satisfy important requirements: (1) It should be close to Λ CDM but yet distinguishable from it, (2) Theory should be free from ghost and tachyon instabilities and (3) Theory should not conflict with local physics. The third requirement is often very stringent and its compliance needs the invoking of special mechanisms. The modified theories necessarily include additional scalar degree(s) of freedom in some form or the other. And to be relevant to late time cosmic acceleration, it should be light mass entity which, on the other hand, could cause a havoc as a fifth force never seen in the laboratory or in the solar neighborhood. It therefore necessary to screen out the effect of the fifth force in a delicate manner.

Broadly, there are two methods of mass screening, the chameleon mechanism and the Vainshtein effect. The chameleon scenario [9] relies on the direct coupling of matter to scalar field with potential. The mass of the field becomes dependent on the density of environment which justifies the designation ”chameleon” for such a field. The chameleon potential is chosen such that the effective mass of the field increases with local matter density thereby leading to suppression of fifth force. The chameleon mechanism is an extremely powerful tool for mass screening but as noticed by many authors (see e.g.

[10]) the numerical integration of the system for a spherically symmetric background hits singularity because of the form of the chameleon potential. Hence an extreme fine-tuning of the initial conditions is required to avoid the problem. The chameleon theories are also plagued with the problem of large quantum corrections because of the large mass of the chameleon field required to pass the local gravity tests [11].

The Vainshtein mechanism [12] is a superior field theoretic method of mass screening. It was invented by Vainshtein in 1972 to address the discontinuity problem in massive gravity of Pauli-Fierz. It relies on non-linear derivative term of the type $L_3 = (\partial_\mu\phi)^2\square\phi$ where ϕ is scalar degree of freedom of helicity zero graviton in this case. The dynamics of non-linear term gives rise to a miraculous phenomenon: Around a massive body, in a large radius dubbed Vainshtein radius, fifth force is suppressed switching off any modification to gravity locally. The field is strongly coupled to itself and hence becomes weakly coupled to matter sector. The DGP model [13] contains such a non-linear term in the so called decoupling limit responsible for the compliance of the model with local gravity constraints [14]. The non-standard kinetic term also occurs in Kaluza-Klein reduction of Gauss-Bonnet gravity to four dimensional space time [15] which makes clear why the theory is free from Ostrogradsky ghosts. The role of the scalar degree of freedom is played by dilaton in this case. The field ϕ due to the presence of an underlying symmetry in flat space time was termed as galileon. There exist higher order galileon Lagrangians L_4 and L_5 that contain higher order non linear derivative terms in which four and five galileon fields participate respectively. Recently, more general galileon action were constructed [16, 17] and their cosmological implications were investigated [18]. They belong to a

more general class of models first introduced in [19].

The galileon field with the lower order term L_3 is sufficient to take care of the local gravity constraints whereas L_5 does not contribute to mass screening and though L_4 can effect the numerics of Vainshtein radius but adds nothing to the underlying physics of Vainshtein mechanism.

However, galileon system with L_3 term alone can not give rise to late time acceleration of universe. It was first demonstrated in [20] that at least one higher order Lagrangian, say L_4 [23] be added to action in order to produce a stable de Sitter solution. This is the analogue of the DGP model, where the self-accelerating branch is unstable because of the presence of ghost [24]. Also physical implications of the cubic Galileon term coupled non-minimally to the metric was first studied in [21, 22]. Since we are working in a phenomenological setting, we could restrict ourselves to L_3 but add a potential to galileon to produce late time cosmic acceleration. We do not assign a field theoretic mechanism to produce the potential for galileon, perhaps one would need some non-perturbative machinery to do the job.

Our proposal can be seen as an attempt to replace chameleon mechanism by the Vainshtein effect: In chameleon scenario, the choice of specific form of chameleon potential is crucial to the model which needs to be extremely fine-tuned locally for the successful implementation of the underlying chameleon ideology.

II. BASIC SETUP AND EQUATIONS OF MOTION

It is well known that in General theory of Relativity, the Newtonian and longitudinal gravitational potentials are equal provided that anisotropic stress tensor is absent. Modified gravity theories usually predict a difference between these two potentials – a gravitational slip. At the linear level of perturbations, the modified theories of gravity give rise to an anisotropic stress tensor which is really not a signature of a particular modified theory as usually claimed. At the linear level of perturbations, the modified theories of gravity give rise to an anisotropic stress tensor which is really not a distinctive signature of modification of gravity as usually claimed. A modified theory of gravity can, in general, be described in terms of an imperfect fluid giving rise to a gravitational slip. Thus we can not distinguish whether the slip is caused by the presence of an imperfect fluid or by the modification of gravity. Since the observations, at present, do not show any difference between the gravitational potentials (see e.g. [25]), it is interesting to study models that do not produce a gravitational slip.

Following the results derived in [26], we deduce that the most general action with second order differential equation for the metric and the scalar field which do not produce a slip is of one of the following form

$$\begin{aligned} \mathcal{S}_1 = \int d^4x \sqrt{-g} & \left[F(\phi)R + \left(F'(\phi) \ln X + G(\phi) \right) \square\phi \right. \\ & + H(\phi) \left((\square\phi)^2 - \phi_{;\mu\nu} \phi^{;\mu\nu} + XR - \frac{3}{2} \nabla_\mu (\phi^{;\mu} \square\phi) \right. \\ & \left. \left. - G_{\mu\nu} \phi^{;\mu} \phi^{;\nu} \right) + K(\phi, X) \right] \end{aligned} \quad (1)$$

or

$$\begin{aligned} \mathcal{S}_2 = \int d^4x \sqrt{-g} & \left[R + G(\phi, X) \square\phi + K(\phi, X) \right. \\ & \left. + F(\phi) \left((\square\phi)^2 - \phi_{;\mu\nu} \phi^{;\mu\nu} + XR - G_{\mu\nu} \phi^{;\mu} \phi^{;\nu} \right) \right] \end{aligned} \quad (2)$$

where $X = -\frac{1}{2} g^{\mu\nu} \phi_{;\mu} \phi_{;\nu}$.

The two actions are different but in both cases we do not have a gravitational slip. The second action \mathcal{S}_2 is a generalization of the KGB model studied in [27]. We also note that for \mathcal{S}_1 in the Brans-Dicke form ϕR , the presence of $\ln X \square\phi$ is important to cancel the slip between the two potentials.

Keeping in mind a well motivated model which appears in the decoupling limit of DGP, we shall consider the second action hereafter. We also introduce a coupling of field to matter, therefore we work in the Einstein frame where the theory does not give rise to a slip. Obviously in the Jordan frame the same theory would produce a slip because of the conformal factor as noticed in [28]. The conformal transformation would produce terms which are not in the form of the original action.

The second action is the most general action free from Ostrogradsky ghost problem, it does not produce a gravitational slip and contains the decoupling limit of DGP as a sub-class. But the first action can be seen as a modified Brans-Dicke model which can produce interesting local and cosmological solutions.

In what follows, we shall consider a simple model with $G \equiv X$ which corresponds to the decoupling limit of DGP, $F \equiv 0$ and $K \equiv X - V(\phi)$. With these choices, our action has the form (in Einstein frame)

$$\begin{aligned} S = \int d^4x \sqrt{-g} & \left[\frac{M_{\text{pl}}^2}{2} R - \frac{1}{2} (\nabla\phi)^2 \left(1 + \frac{\alpha}{M^3} \square\phi \right) - V(\phi) \right] \\ & + \mathcal{S}_m \left[\psi_m; e^{2\beta\phi/M_{\text{pl}}} g_{\mu\nu} \right] \end{aligned} \quad (3)$$

where $M_{\text{pl}}^2 = 8\pi G$ is the reduced Planck mass, M is a energy scale, (α, β) are dimensionless constants and V is the potential for the field.

This action corresponds to the coupled quintessence field with a galileon like correction $(\nabla\phi)^2 \square\phi$. Variation of the action (3) gives the following equations of motion

$$M_{\text{pl}}^2 G_{\mu\nu} = T_{\mu\nu}^{(m)} + T_{\mu\nu}^{(r)} + T_{\mu\nu}^{(\phi)} \quad (4)$$

$$\begin{aligned} \square\phi + \frac{\alpha}{M^3} & \left[(\square\phi)^2 - \phi_{;\mu\nu} \phi^{;\mu\nu} - R^{\mu\nu} \phi_{;\mu} \phi_{;\nu} \right] \\ - V'(\phi) & = -\frac{\beta}{M_{\text{pl}}} T^{(m)} \end{aligned} \quad (5)$$

where

$$T_{\mu\nu}^{(\phi)} = \phi_{;\mu}\phi_{;\nu} - \frac{1}{2}g_{\mu\nu}(\nabla\phi)^2 - g_{\mu\nu}V(\phi) + \frac{\alpha}{M^3} \left[\phi_{;\mu}\phi_{;\nu}\square\phi + g_{\mu\nu}\phi_{;\lambda}\phi^{;\lambda\rho}\phi_{;\rho} - \phi_{;(\mu}\phi_{;\nu)\rho}\phi^{;\rho} \right] \quad (6)$$

where ' denotes the derivative wrt ϕ .

In the discussion to follow, we shall investigate the cosmological dynamics and local gravity constraints in the model based upon the equations of motion (5).

III. FIFTH FORCE, SCREENING EFFECT AND LOCAL PHYSICS

In our model, in general, we might have both the screening mechanisms operating due to the presence of potential and the galileon term. We wish to arrange things in such a way that potential contributes negligibly to local physics giving way to Vainshtein mechanism. The potential should come into action at large scales giving rise to cosmic acceleration.

We now consider spherically symmetric, static solution of equation (5). In flat Minkowski background and in presence of non relativistic matter, we have

$$\frac{1}{r^2} \frac{d}{dr} [r^2 \phi'(r)] + \frac{2\alpha}{M^3 r^2} \frac{d}{dr} [r \phi'(r)^2] - \frac{dV(\phi)}{d\phi} = \frac{\beta}{M_{pl}} \rho_m \quad (7)$$

We consider that the same phenomenon is at the origin of late time acceleration and local suppression of fifth force in the action. We therefore associate to both the non-linear term and the potential the same energy scale M . We have $\phi \simeq M$ and $V \simeq M^4$. As we previously mentioned, the chameleon mechanism generates huge fine-tuning conditions. Thus if we want to prefer the Vainshtein mechanism, we should demand that at small scales $\frac{2\alpha}{M^3 r^2} \frac{d}{dr} [r \phi'(r)^2] \gg \frac{dV(\phi)}{d\phi}$. Then considering $\alpha \simeq 1$ we may have a negligible potential contribution for scales $L \ll 1/M$. This scale should at least be larger than the apogee which tells us that $M \ll 10^{-15} \text{eV}$. Also we should assume that the potential term is negligible compared to the matter part. Considering $\rho_m \simeq 1 \text{g/cm}^{-3}$, we have $M \ll 10^{-3} \text{eV}$. This condition could always be verified as soon as the first one is fixed. We shall see later that for viable cosmology, the potential should be of the order of critical density which implies $M \simeq 10^{-3} \text{eV}$. It seems contradictory with the previous statement. In fact, from the energy scale M , the results will also depend on the form of the potential. We found that for inverse power law potentials, we are in the chameleon regime and hence divergences can occur. We will take up this issue in the next section where we shall consider the class of potentials in detail which give negligible contribution locally for $M \simeq 10^{-3} \text{eV}$.

In that case, neglecting the potential, the system reduces to the standard Galileon with L_3 . At large scales,

the potential is dominant and this part would be studied in the next section. We note there is also an intermediate regime where the potential is of the order of L_3 . We would not consider the analysis of this regime here. In fact at very large distances, the approximations made in this section can not be trusted, the expansion of the Universe should be taken into account too. Last but not least, as there is no physical system isolated at large scales, it is not known how the screening mechanism would operate for N particles if each one of them possess its own Vainshtein radius – the so-called *elephant* problem.

For the scales of interest mentioned above, the fifth force is

$$F_\phi = \frac{\beta M^3 r}{4\alpha M_{pl}} \left[-1 + \sqrt{1 + \left(\frac{r_V}{r} \right)^3} \right] \quad (8)$$

where r_V is the Vainshtein radius of the body with mass M and r_s is the Schwarzschild radius.

$$r_v = \left(\frac{8\alpha\beta M_{pl} r_s}{M^3} \right)^{\frac{1}{3}} \quad (9)$$

The Vainshtein radius is defined as the distance from where onwards the fifth force becomes comparable with the gravitational force. Considering the Sun, we need a Vainshtein radius larger than the size of solar system which implies that $M < 10^{-7}(\alpha\beta)^{1/3} \text{eV}$. The scale M plays a fundamental role in local physics determined by Vainshtein mechanism. This is also the scale associated with the potential thereby to the cosmology of the model. In order to get late time acceleration of the Universe, we need to fix, $M \simeq 10^{-3} \text{eV}$. On the other hand, in order to have Vainshtein radius larger than the solar system, we should demand that $\alpha\beta > 10^{12}$. Another option would be to choose $\alpha \simeq 1$ or $M^4 \ll \Lambda$ where Λ is the cosmological constant. This would make the model even less natural than the cosmological constant in terms of an unnatural value. Perhaps, it points towards a possibility that the scale could be linked to some large extra dimension *à la* DGP model. Finally, one could also consider two different scales (M_1, M_2) in the model associated with the galileon term and the potential respectively. We shall not consider this possibility here.

In our analysis we have considered two different potentials

$$V(\phi) = M^4 \left[e^{-\frac{\mu_1 \phi}{M_{pl}}} + e^{-\frac{\mu_2 \phi}{M_{pl}}} \right] \quad (10)$$

and

$$V(\phi) = M^4 \left[\cosh \left(\frac{\gamma \phi}{M_{pl}} \right) - 1 \right]^m, \quad (11)$$

where λ, μ, γ are constants and m is a positive integer. In both cases, we found numerically that the potential is negligible locally and that the eq.(7) can be approximated by a vanishing potential.

The system appears as the decoupling limit of DGP and as it is well known that the Vainshtein mechanisms protects the model in a static spherically symmetric configuration. In the next section we shall demonstrate that the potential is dominant at large scales and gives rise to a correct cosmology contrary to the same model without potential [20].

IV. ON THE BOUND FROM THE TIME VARIATION OF G

All the theories described in the Einstein frame by a time-varying coupling to matter leads to a variation of the effective Newton constant (G) as seen in the Jordan frame. There are two effects which modify G [29]: The exchange of helicity-0 modes which is suppressed in our model because of the Vainshtein mechanism and the rescaling of the coordinates because of the conformal transformation. In our case the time variation of G occurs because of the rescaling factor only. Hence it is easy to see that

$$|\dot{G}/G| \approx \beta \dot{\phi}/M_{pl}. \quad (12)$$

It was shown in Ref.[29] that a model involving only derivatives of the fields would give

$$|\dot{G}/G|_{\text{today}} \approx \beta H_0. \quad (13)$$

But the observational constraints from the Lunar Laser Ranging (LLR) gives $|\dot{G}/G|_{\text{today}} < 0.01 H_0$ [30] which translates into a restriction on β , namely, $\beta < 0.01$. In the presence of a potential, the situation is drastically modified. In fact, in this case, we can assume that the field sits at the minimum of the effective potential. As a result, in case of the potential (10), we have

$$|\dot{G}/G|_{\text{today}} \approx \frac{6\beta^2}{\beta^2 + \mu_1^2 + \mu_2^2} H_0 \quad (14)$$

Hence the LLR constraint gives

$$\beta \lesssim \frac{\sqrt{\mu_1^2 + \mu_2^2}}{25} \quad (15)$$

In the next section, we will consider cosmology for ($\mu_1 = 20 \gg \mu_2$) which means that $\beta \leq 1$. Therefore we see that the LLR bound will not bring additional constraints as soon as the standard cosmological constraints are satisfied.

V. COSMOLOGICAL DYNAMICS OF THE MODEL

In a spatially flat FLRW background, the equations of motion take the form

$$3M_{\text{pl}}^2 H^2 = \rho_m + \rho_r + \frac{\dot{\phi}^2}{2} \left(1 - \frac{6\alpha}{M^3} H \dot{\phi}\right) + V(\phi), \quad (16)$$

$$M_{\text{pl}}^2 (2\dot{H} + 3H^2) = -\frac{\rho_r}{3} - \frac{\dot{\phi}^2}{2} \left(1 + \frac{2\alpha}{M^3} \ddot{\phi}\right) + V(\phi), \quad (17)$$

$$-\frac{\beta}{M_{\text{pl}}} \rho_m = \ddot{\phi} + 3H\dot{\phi} - \frac{3\alpha}{M^3} \dot{\phi} (3H^2 \dot{\phi} + \dot{H}\dot{\phi} + 2H\ddot{\phi}) + V'(\phi). \quad (18)$$

The equation of conservation which can be derived from the previous equations is

$$\dot{\rho}_m + 3H\rho_m = \frac{\beta}{M_{\text{pl}}} \dot{\phi} \rho_m, \quad (19)$$

$$\dot{\rho}_r + 4H\rho_r = 0 \quad (20)$$

Let us introduce the following dimensionless quantities

$$x = \frac{\dot{\phi}}{\sqrt{6}HM_{\text{pl}}}, \quad y = \frac{\sqrt{V}}{\sqrt{3}HM_{\text{pl}}} \quad (21)$$

$$\epsilon = -6\frac{\alpha}{M^3} H \dot{\phi}, \quad \lambda = -M_{\text{pl}} \frac{V'}{V} \quad (22)$$

needed to cast the evolution equations in the form of an autonomous system

$$\frac{dx}{dN} = x \left(\frac{\ddot{\phi}}{H\dot{\phi}} - \frac{\dot{H}}{H^2} \right) \quad (23)$$

$$\frac{dy}{dN} = -y \left(\sqrt{\frac{3}{2}} \lambda x + \frac{\dot{H}}{H^2} \right) \quad (24)$$

$$\frac{d\epsilon}{dN} = \epsilon \left(\frac{\ddot{\phi}}{H\dot{\phi}} + \frac{\dot{H}}{H^2} \right) \quad (25)$$

$$\frac{d\Omega_r}{dN} = -2\Omega_r \left(2 + \frac{\dot{H}}{H^2} \right) \quad (26)$$

$$\frac{d\lambda}{dN} = \sqrt{6}x\lambda^2(1 - \Gamma) \quad (27)$$

where $N \equiv \ln a$, $\Gamma = \frac{VV_{,\phi\phi}}{V_{,\phi}^2}$ and

$$\frac{\dot{H}}{H^2} = \frac{2(1 + \epsilon)(-3 + 3y^2 - \Omega_r) - 3x^2(2 + 4\epsilon + \epsilon^2)}{4 + 4\epsilon + x^2\epsilon^2} + \frac{\sqrt{6}x\epsilon(y^2\lambda - \beta\Omega_m)}{4 + 4\epsilon + x^2\epsilon^2} \quad (28)$$

$$\frac{\ddot{\phi}}{H\dot{\phi}} = \frac{3x^3\epsilon - x(12 + \epsilon(3 + 3y^2 - \Omega_r)) + 2\sqrt{6}(y^2\lambda - \beta\Omega_m)}{x(4 + 4\epsilon + x^2\epsilon^2)} \quad (29)$$

In the case of an exponential form of the potential, $\Gamma = 1$ and λ is a constant. Therefore the system reduces to the set of four equations as usual.

For $\epsilon = 0$, we recover the standard coupled dark-energy model [31]. The system has the same dynamical phase plane as the coupled quintessence except one additional de-Sitter solution, for $(\epsilon = -2, \lambda = 0, y^2 - x^2 = 1)$. This solution exists only if the equation $\lambda^2 \Gamma(\lambda) = 0$ admits the solution $\lambda = 0$. Before getting to numerical analysis, we should broadly summarize the constraints on the model parameters. we have parameters (μ_1, μ_2) in the potential (10) which control the different phases of the dynamics. One of the parameters μ_1 , gives rise to the radiation era whereas, μ_2 is responsible for the late time cosmic acceleration. The matter era is principally controlled by β . In order to have viable thermal history ($\Omega_r = 1 - 4/\mu_1^2$), we need to impose a constraint on μ_1 . Indeed, the nucleosynthesis demands that $\Omega_\phi = 4/\mu_1^2 \leq 0.01$ during the radiation era which imposes restriction on μ_1 , namely, $\mu_1 \geq 20$. For stable acceleration of the Universe we need to impose constraints on other parameters such that, $\mu_2 < \sqrt{2}$ and also $\beta < (3 - \mu_2^2)/\mu_2$. During the Dark Energy phase, $\omega_{\text{eff}} = -1 + \mu_2^2/3$. We shall make use of this information while carrying out the numerical analysis. Indeed, in the numerics to follow, we will take small values for β and adhere to $(\mu_1, \mu_2) = (20, 0.5)$. During the evolution, λ would change from μ_1 to μ_2 hence if μ_2 is small enough, it would imply a small variation of the potential which would give rise to quasi-de-Sitter universe, $\omega_{\text{eff}} \simeq -1$.

We have numerically analysed the autonomous system and find that the cosmology of the model is similar to the case of coupled quintessence Fig.(1,2). The influence of the higher derivative term disappears fast with evolution leaving behind the coupled quintessence. We have not listed the various fixed points and their corresponding eigenvalues to avoid the repetition of the earlier work on the similar theme. We have plotted the dimensionless density parameter and field energy density versus the scale factor on the log scale for the double exponential potential. The plots show in case of the overshoot a tracker behaviour. The parameters can be conveniently fixed to produce a viable late time cosmology.

VI. COSMOLOGICAL PERTURBATION

In this section we analyse the perturbations of the model. As mentioned in the introduction, the action is built in such away that the two gravitational potentials are same. Hence we consider the following metric

$$ds^2 = -(1 + 2\Psi)dt^2 + a(t)^2(1 - 2\Psi)d\vec{x}^2 \quad (30)$$

In the subhorizon approximation, we have

$$\ddot{\delta}_m + (2H + \frac{\beta}{M_{pl}}\dot{\phi})\dot{\delta}_m - 4\pi G_{\text{eff}}\rho_m\delta_m = 0 \quad (31)$$

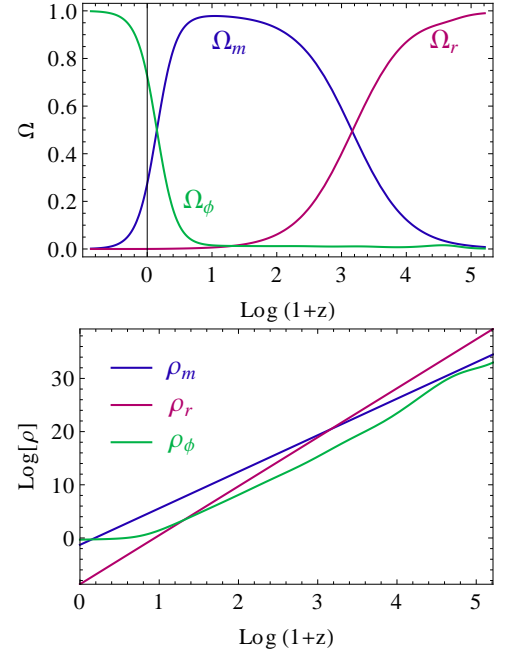


FIG. 1: (Top): Density parameters Ω of matter, radiation and galileon field for potential (10) are shown here with $\mu_1 = 20$, $\mu_2 = 0.5$ and $\beta = 0.1$. (Bottom): Density $\log\left(\frac{\rho}{3M_{pl}^2 H_0^2}\right)$ for the 3 fields and the same parameters (μ_1, μ_2, β) .

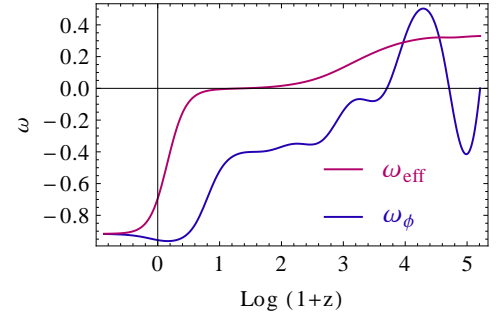


FIG. 2: The cosmic evolution of the field equation of state w_ϕ and the total effective equation of state w_{eff} for $\mu_1 = 20$, $\mu_2 = 0.5$ and $\beta = 0.1$.

Where,

$$G_{\text{eff}} = G \left(1 + \frac{\left(\frac{\alpha}{M^3 M_{pl}} \dot{\phi}^2 + 2\beta \right)^2}{2 - \frac{\alpha^2}{M^6 M_{pl}^2} \dot{\phi}^4 - 8 \frac{\alpha}{M^3} H \dot{\phi} - 4 \frac{\alpha}{M^3} \ddot{\phi}} \right) \quad (32)$$

where the gauge-invariant density contrast is defined as $\delta_m = \frac{\delta \rho_m}{\rho_m} + 3Hv$ and v is the velocity of the fluid.

In Fig.(3), we show the evolution of G_{eff} as a function of the redshift. Because of the non-linear term of the galileon field, we have at large redshift G_{eff} similar to General Relativity which is consistent with BBN constraints, whereas $G_{\text{eff}} = G(1 + 2\beta^2) > G$ in the coupled quintessence model. Thus the non-linear term of

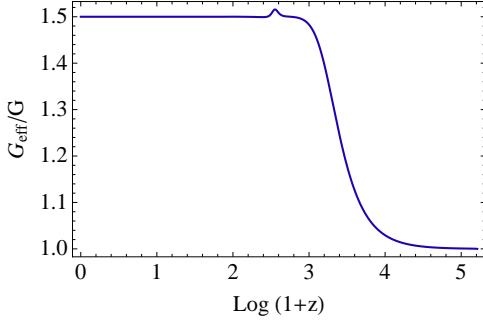


FIG. 3: Evolution of G_{eff}/G for the potential (10) with $\mu_1 = 20$, $\mu_2 = 0.5$ and $\beta = 0.5$.

the galileon field can alleviate constraints on the model. We have seen that the bound from the LLR is weak, hence we should expect to have large values of the coupling β . However, we shall notice in the section to follow that the model under consideration reduces to coupled quintessence scenario at large scales thereby giving rise to strong cosmological constraints on β as first noticed in Ref.[31].

VII. DATA ANALYSIS

To get some insight into the parameter window for the models, we constrain them using Supernovae, BAO and CMB data. We have also used the data of the growth factor f [32] defined as

$$f = \frac{d \ln \delta_m}{d \ln a} \quad (33)$$

We show the evolution of the growth factor f for the two potentials studied in this paper. In both cases, the deviation from Λ CDM model is not significant. We should also mention that $f > 1$ at $z = 3$ which means that the growth is faster than in Einstein-de-Sitter model. This is opposite to the prediction of Dark Energy scenario. This sounds very strange but since the deviation from Λ CDM is not significant, the latter does not effect the statistical analysis. Only future observations can confirm this result.

We define

$$\chi^2 = \chi_{\text{Growth}}^2 + \chi_{\text{SN}}^2 + \chi_{\text{BAO}}^2 + \chi_{\text{CMB}}^2. \quad (34)$$

where χ_{Growth}^2 is defined as,

$$\chi_{\text{Growth}}^2(\theta) = \sum_i \frac{(f_{\text{obs}}(z_i) - f_{\text{th}}(z_i, \theta))^2}{\sigma_f(z_i)^2} \quad (35)$$

Where, f_{obs} is the observational data and σ_f is the 1σ error. f_{th} is the theoretically calculated value of growth factor from the eqn(33). θ is the model parameter.

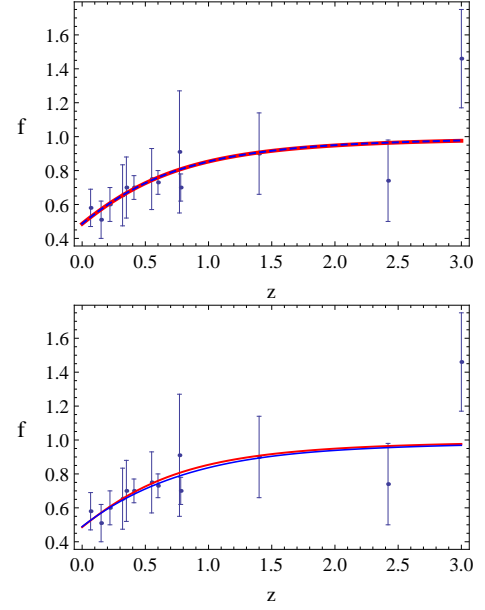


FIG. 4: Evolution of the growth factor f for potential (10) and (11) respectively. The integration is done from $\Omega_m \approx 0.98$ for both the cases with $\beta = 0.1$. For the potential (10) we have taken $\mu_1 = 20$ and $\mu_2 = 0.5$ and for the potential (11) we have taken $\gamma = 30$ and $m = 0.5$. The red line corresponds to Λ CDM model.

For the SN type Ia observation we consider Union2.1 data compilation [33] of 580 data points. The observable for the SN type Ia is the distance modulus μ which is defined as, $\mu = m - M = 5 \log D_L + \mu_0$, where m and M are the apparent and absolute magnitudes of the Supernovae and luminosity distance $D_L(z) = (1+z) \int_0^z \frac{H_0 dz'}{H(z')}$ and $\mu_0 = 5 \log \left(\frac{H_0^{-1}}{M_{pc}} \right) + 25$ is a nuisance parameter and should be marginalized. For this observation the χ^2 is defined as,

$$\chi_{\text{SN}}^2(\mu_0, \theta) = \sum_{i=1}^{580} \frac{(\mu_{\text{th}}(z_i, \mu_0, \theta) - \mu_{\text{obs}}(z_i))^2}{\sigma_\mu(z_i)^2} \quad (36)$$

Where, σ_μ is the uncertainty in the distance modulus. After marginalizing μ_0 as [34] we can have,

$$\chi_{\text{SN}}^2(\theta) = A - \frac{B^2}{C} \quad (37)$$

Where,

χ_{BAO}^2 is defined as,

$$A(\theta) = \sum_{i=1}^{580} \frac{(\mu_{th}(z_i, \mu_0, \theta) - \mu_{obs}(z_i))^2}{\sigma_\mu(z_i)^2} \quad (38)$$

$$B(\theta) = \sum_{i=1}^{580} \frac{\mu_{th}(z_i, \mu_0, \theta) - \mu_{obs}(z_i)}{\sigma_\mu(z_i)^2} \quad (39)$$

$$C(\theta) = \sum_{i=1}^{580} \frac{1}{\sigma_\mu(z_i)^2} \quad (40)$$

For BAO we have used the data of $\frac{d_A(z_*)}{D_V(Z_{BAO})}$ [35], where z_* is the decoupling time $z_* \approx 1091$, the co-moving angular-diameter distance $d_A(z) = \int_0^z \frac{dz'}{H(z')}$ and $D_V(z) = \left(d_A(z)^2 \frac{z}{H(z)}\right)^{\frac{1}{3}}$. Required data for this analysis is given in table I.

$$\chi_{BAO}^2 = X_{BAO}^T C_{BAO}^{-1} X_{BAO} \quad (41)$$

Where,

$$X_{BAO} = \begin{pmatrix} \frac{d_A(z_*)}{D_V(0.106)} - 30.95 \\ \frac{d_A(z_*)}{D_V(0.2)} - 17.55 \\ \frac{d_A(z_*)}{D_V(0.35)} - 10.11 \\ \frac{d_A(z_*)}{D_V(0.44)} - 8.44 \\ \frac{d_A(z_*)}{D_V(0.6)} - 6.69 \\ \frac{d_A(z_*)}{D_V(0.73)} - 5.45 \end{pmatrix} \quad (42)$$

and the inverse covariance matrix,

$$C^{-1} = \begin{pmatrix} 0.48435 & -0.101383 & -0.164945 & -0.0305703 & -0.097874 & -0.106738 \\ -0.101383 & 3.2882 & -2.45497 & -0.0787898 & -0.252254 & -0.2751 \\ -0.164945 & -2.45499 & 9.55916 & -0.128187 & -0.410404 & -0.447574 \\ -0.0305703 & -0.0787898 & -0.128187 & 2.78728 & -2.75632 & 1.16437 \\ -0.097874 & -0.252254 & -0.410404 & -2.75632 & 14.9245 & -7.32441 \\ -0.106738 & -0.2751 & -0.447574 & 1.16437 & -7.32441 & 14.5022 \end{pmatrix}. \quad (43)$$

Finally we used the CMB shift parameter $R = H_0 \sqrt{\Omega_{m0}} \int_0^{1089} \frac{dz'}{H(z')}$. For this χ_{CMB}^2 is defined as,

$$\chi_{CMB}^2(\theta) = \frac{(R(\theta) - R_0)^2}{\sigma^2} \quad (44)$$

Where, $R_0 = 1.725 \pm 0.018$ [36].

In this analysis we have two model parameters β and Ω_{m0} . We varied β from -0.5 to 1 and Ω_{m0} from 0.2 to 0.33 . Figure (5) shows the results in the (β, Ω_{m0}) parameter space. The analysis is performed for $M = 10^{-3}$ eV. Minimum value of the total χ^2 is at $\beta \sim 0.04$ and $\Omega_{m0} \sim 0.275$. Using these best fit values of the model parameters growth index γ is plotted in the figure (6) with the 1σ and 2σ errors. At late time, the evolution is consistent with Dark Energy models where $w > -1$ [37]. But the model shows a larger value of γ for all redshift compared to Λ CDM. This is a different characteristic than $f(R)$ -gravity models [38] or scalar-tensor theories [39] where $\gamma < 0.55$.

We find that $\beta \simeq 0$, β large is not a problem for local physics (chameleon, Vainshtein) but at large scales we need to consider β small as in coupled quintessence. In fact, the galileon term is negligible at large scales and we recover the standard coupled quintessence model.

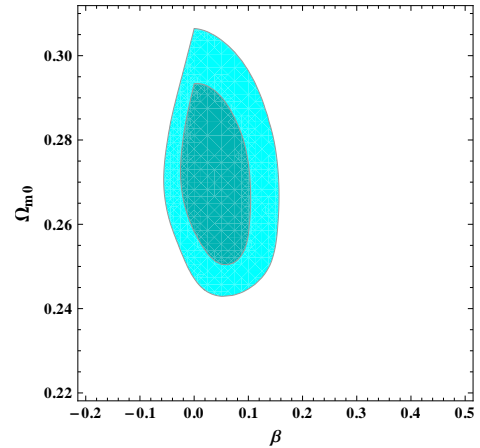


FIG. 5: The 1σ (inside) and 2σ (outside) likelihood contours in the $(\Omega_{m,0}, \beta)$ plane for total $\chi_{Growth+SN+BAO+CMB}^2$.

VIII. CONCLUSION

In this paper, we have investigated a particular case of a more general class of models which do not produce a gravitational slip. This model includes the L_3 -term derived in the decoupling limit of DGP and a galileon field

z_{BAO}	0.106	0.2	0.35	0.44	0.6	0.73
$\frac{d_A(z_*)}{D_V(z_{BAO})}$	30.95 ± 1.46	17.55 ± 0.60	10.11 ± 0.37	8.44 ± 0.67	6.69 ± 0.33	5.45 ± 0.31

TABLE I: Values of $\frac{d_A(z_*)}{D_V(z_{BAO})}$ for different values of z_{BAO} .

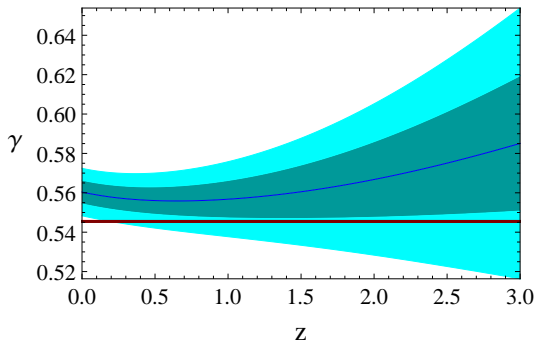


FIG. 6: The 1σ (inside) and 2σ (outside) of the growth index. The central line is the growth index with the best fit values of the model parameters β and Ω_{m0} in the case of the double exponential (10).

potential added to the Lagrangian on phenomenological grounds. The potential breaks the galilean symmetry in a flat spacetime but serves as an important toll at large scales for obtaining a viable cosmology. The scale in the model and the parameters are carefully set such that the galileon term is dominant at small scales which suppress the effect of the chameleon effect leaving the Vainshtein mechanism to operate. This was one of the motives of our proposal to replace the chameleon mechanism as the latter is plagued by the problems of fine tuning and large quantum corrections due to the large mass of the chameleon field. In contrast, the domination of the galileon term at small scales successfully generates a screening effect without requiring a large mass of the field and the quantum corrections remain small. We have, therefore shown that models including light mass scalar fields coupled to matter become viable as soon as we introduce a lower order galileon, term, $(\nabla\phi)^2\Box\phi$ in

the action. We should also emphasize that this term does not significantly modify the background cosmology of the model. Our model is built in away as not to produce large deviation from standard model for cosmological perturbations due to the vanishing of the gravitational slip. This approach can be generalized to a large class of light mass scalar field models which are otherwise ruled out by local gravitational constraints.

We have demonstrated that the model passes the local tests and can give rise to viable cosmology at late times. It produces a small deviation of the growth factor compared to Λ CDM model. It interesting to note that in case of pure coupled quintessence, $G_{eff} = G(1 + 2\beta^2)$ thereby requiring coupling to be small in order to respect the BBN constraint. On the other hand, in the model under consideration, $G_{eff} = G$ as in General Relativity which certainly alleviates the constraints on coupling at large redshifts. In principle, our scenario involves cosmological constraints at low redshift which depends on the form of the potential as usual.

IX. ACKNOWLEDGEMENTS

The work of RG is supported by the Grant-in-Aid for Scientific Research Fund of the JSPS No. 10329. MWH acknowledges the funding from CSIR, Govt of India. MWH wants to thank Sumit Kumar for useful discussions on statistical techniques. MS thanks S. Nojiri and K. Bamba for useful discussions. MS is supported by JSPS fellowship and thanks Kobayashi-Maskawa Institute for the Origin of Particles and the Universe for hospitality. MS is also supported by the Department of Science and Technology, India under project No. SR/S2/HEP-002/2008.

-
- [1] V. Sahni and A. A. Starobinsky, Int. J. Mod. Phys. D **9**, 373 (2000).
 - [2] V. Sahni and A. Starobinsky, Int.J.Mod.Phys.D **15**, 2105(2006)[astro-ph/0610026]; T. Padmanabhan, astro-ph/0603114; P. J. E. Peebles and B. Ratra, Rev. Mod. Phys. **75**, 559 (2003); L. Perivolaropoulos, astro-ph/0601014; N. Straumann, arXiv:gr-qc/0311083; J. Frieman, arXiv:0904.1832; M. Sami, Lect. Notes Phys.**72**, 219(2007); M. Sami, arXiv:0901.0756 .
 - [3] E. J. Copeland, M. Sami and S. Tsujikawa, Int. J. Mod. Phys., D15 , 1753(2006)[hep-th/0603057].
 - [4] E. V. Linder, Rep. Prog. Phys. **71** (2008) 056901.
 - [5] R. R. Caldwell and M. Kamionkowski, Ann. Rev. Nucl. Part. Sci. **59** (2009) 397 [arXiv:0903.0866 [astro-ph.CO]].
 - [6] A. Silvestri and M. Trodden, Rept. Prog. Phys. **72** (2009) 096901 [arXiv:0904.0024 [astro-ph.CO]].
 - [7] J. Frieman, M. Turner and D. Huterer, Ann. Rev. Astron. Astrophys. **46** (2008) 385 [arXiv:0803.0982 [astro-ph]].
 - [8] M. Sami, Curr. Sci. **97** (2009) 887 [arXiv:0904.3445 [hep-th]].
 - [9] J. Khoury and A. Weltman, Phys. Rev. Lett. **93** (2004) 171104 [astro-ph/0309300]; J. Khoury and A. Weltman, Phys. Rev. D **69** (2004) 044026 [astro-ph/0309411].
 - [10] E. Babichev and D. Langlois, Phys. Rev. D **81** (2010) 124051 [arXiv:0911.1297 [gr-qc]];
 - [11] D.F. Mota, Jan Øye Lindroos: Private communication.
 - [11] A. Upadhye, W. Hu and J. Khoury, arXiv:1204.3906

- [hep-ph];
R. Gannouji, M. Sami and I. Thongkool, arXiv:1206.3395 [hep-th].
- [12] A. I. Vainshtein, Phys. Lett. B **39** (1972) 393.
- [13] G. R. Dvali, G. Gabadadze and M. Porrati, Phys. Lett. B **485** (2000) 208 [hep-th/0005016].
- [14] M. A. Luty, M. Porrati and R. Rattazzi, JHEP **0309** (2003) 029 [hep-th/0303116].
- [15] R. Gannouji and M. Sami, Phys. Rev. D **85** (2012) 024019 [arXiv:1107.1892 [gr-qc]].
- [16] C. de Rham and A. J. Tolley, JCAP **1005** (2010) 015 [arXiv:1003.5917 [hep-th]].
- [17] G. Goon, K. Hinterbichler and M. Trodden, JCAP **1107** (2011) 017 [arXiv:1103.5745 [hep-th]].
- [18] A. De Felice and S. Tsujikawa, Phys. Rev. Lett. **105** (2010) 111301 [arXiv:1007.2700 [astro-ph.CO]];
A. De Felice, R. Kase and S. Tsujikawa, Phys. Rev. D **83** (2011) 043515 [arXiv:1011.6132 [astro-ph.CO]];
T. Qiu, J. Evslin, Y. -F. Cai, M. Li and X. Zhang, JCAP **1110** (2011) 036 [arXiv:1108.0593 [hep-th]];
G. Goon, K. Hinterbichler and M. Trodden, JCAP **1112** (2011) 004 [arXiv:1109.3450 [hep-th]];
S. A. Appleby and E. V. Linder, JCAP **1203** (2012) 043 [arXiv:1112.1981 [astro-ph.CO]];
N. Shirai, K. Bamba, S. Kumekawa, J. Matsumoto and S. i. Nojiri, arXiv:1203.4962 [hep-th].
- [19] D. B. Fairlie, J. Govaerts and A. Morozov, Nucl. Phys. B **373** (1992) 214 [hep-th/9110022];
D. B. Fairlie and J. Govaerts, J. Math. Phys. **33** (1992) 3543 [hep-th/9204074];
D. B. Fairlie, J. Nonlin. Math. Phys. **9** (2002) 256 [math-ph/0112033];
D. Fairlie, J. Phys. A **44** (2011) 305201 [arXiv:1102.1594 [hep-th]].
- [20] R. Gannouji and M. Sami, Phys. Rev. D **82** (2010) 024011 [arXiv:1004.2808 [gr-qc]].
- [21] N. Chow and J. Khoury, Phys. Rev. D **80** (2009) 024037 [arXiv:0905.1325 [hep-th]].
- [22] F. PSilva and K. Koyama, Phys. Rev. D **80** (2009) 121301 [arXiv:0909.4538 [astro-ph.CO]].
- [23] A. Nicolis, R. Rattazzi and E. Trincherini, Phys. Rev. D **79** (2009) 064036 [arXiv:0811.2197 [hep-th]].
- [24] K. Koyama, Phys. Rev. D **72** (2005) 123511 [hep-th/0503191].
- [25] S. F. Daniel, R. R. Caldwell, A. Cooray, P. Serra, A. Melchiorri, A. Cooray, P. Serra and A. Melchiorri, Phys. Rev. D **80** (2009) 023532 [arXiv:0901.0919 [astro-ph.CO]].
- [26] A. De Felice, T. Kobayashi and S. Tsujikawa, Phys. Lett. B **706** (2011) 123 [arXiv:1108.4242 [gr-qc]].
- [27] C. Deffayet, O. Pujolas, I. Sawicki and A. Vikman, JCAP **1010** (2010) 026 [arXiv:1008.0048 [hep-th]].
- [28] P. Brax, R. Rosenfeld and D. A. Steer, JCAP **1008** (2010) 033 [arXiv:1005.2051 [astro-ph.CO]].
- [29] E. Babichev, C. Deffayet and G. Esposito-Farese, Phys. Rev. Lett. **107** (2011) 251102 [arXiv:1107.1569 [gr-qc]].
- [30] J. G. Williams, S. G. Turyshev and D. H. Boggs, Phys. Rev. Lett. **93** (2004) 261101 [gr-qc/0411113].
- [31] L. Amendola, Phys. Rev. D **62** (2000) 043511 [astro-ph/9908023].
- [32] E. Hawkins *et al.*, Mon. Not. Roy. Astron. Soc. **346**, 78 (2003) [arXiv:astro-ph/0212375]; E. V. Linder, arXiv:0709.1113 [astro-ph].
L. Verde *et al.*, Mon. Not. Roy. Astron. Soc. **335**, 432 (2002) [arXiv:astro-ph/0112161].
C. Blake *et al.*, Mon. Not. Roy. Astron. Soc. **415**, 2876 (2001) [arXiv:1104.2948]
R. Reyes *et al.* Nature. **464**, 256, (2010) [arXiv:1003.2185]
M. Tegmark *et al.*, Phys. Rev. D **74**, 123507 (2006) [arXiv:astro-ph/0608632].
N. P. Ross *et al.*, arXiv:astro-ph/0612400.
L. Guzzo *et al.*, Nature. **451**, 541, (2008)
J. da Angela *et al.*, arXiv:astro-ph/0612401.
P. McDonald *et al.* [SDSS Collaboration], Astrophys. J. **635**, 761 (2005) [arXiv:astro-ph/0407377].
F. Beutler *et al.*, (2012), [arXiv:1204.4725]
J. Dosset *et al.*, JCAP **1004**, 022, (2010)
M. de Viel, M. G. Haehnelt and V. Springel, Mon. Not. Roy. Astron. Soc. **354**, 684 (2004)
- [33] N. Suzuki *et al.* 2012 ApJ **746** 85 [arXiv:1105.3470 [astro-ph]]
- [34] R. Lazkoz, S. Nesseris and L. Perivolaropoulos, JCAP **0511** (2005) 010.
- [35] R. Giostri *et al.*, [arXiv:1203.3213 [astro-ph]] and the references mentioned there for BAO data.
- [36] E. Komatsu *et al.*, 2011 ApJS **192** 18
- [37] D. Polarski and R. Gannouji, Phys. Lett. B **660** (2008) 439 [arXiv:0710.1510 [astro-ph]].
- [38] R. Gannouji, B. Moraes and D. Polarski, JCAP **0902** (2009) 034 [arXiv:0809.3374 [astro-ph]].
- [39] R. Gannouji and D. Polarski, JCAP **0805** (2008) 018 [arXiv:0802.4196 [astro-ph]].